Reprinted from the Journal of the Franklin Institute, March, 1905.

Determination of Exponents of Adiabatics.

By Thomas M. Gardner,
Member of the Institute.

LIBRARY OF ILL. URBANA

12Ap1910MB

A paper showing that no simple relationship exists between the values of adiabatic exponents and specific, latent or total heats. Algebraic and graphical methods of determination are used. Tables of average and instantaneous values of *n*, with their corresponding sheets of curves, are given for steam, carbon disulphide, ether, carbon tetrachloride and acetone. Surfaces having instantaneous exponent, pressure and quality values on coordinate axes are shown.—The Editor.

It was found by Rankine that the law of adiabatic expansion of steam was approximately represented by $pv^n = c$, and gave 10/9 as the value of n. A series of pressures with their corresponding volumes were chosen, and the final pressures with their volumes obtained. These values were inserted in the formula:

$$p_1 v_1^n = p_2 v_2^n,$$

or

$$n = (\log. p_1 - \log. p_2) \div (\log. v_2 - \log. v_1).$$

The several results were averaged, and the number given above for steam was obtained. This value of n is only an average exponent; if from $pv^n = c$ the curve be drawn, it will intersect the ideal expansion line in two points, and will differ at all others.

As the exponent varies for different ranges of expansion on the adiabatic, it is desirable that the instantaneous values of n for a number of equal intervals of pressure be determined. Since

$$pv^n = c$$

$$\log. p = n \log. v$$

$$dp / p = n\, dv / v$$

whence

$$dp / dv \div p / v = n.$$

p 19324

REMOTE STORAGE

8367
9G-17d
pam

Method of obtaining the slope dp / dv:

p	v	Δv	$\Delta_2 v$	$\Delta_3 v$
p_1	v_1			
		a_1		
p_2	v_2		b_1	
		a_2		c_1
p_3	v_3		b_2	
		a_3		c_2
p_4	v_4		b_3	
		a_4		c_3
p_5	v_5		b_4	
		a_5		c_4
p_6	v_6		b_5	
		a_6		
p_7	v_7			

Suppose it is desired to find dv / dp at p_4. Beginning at v_4, the first, second and third differences are a_3, b_2 and c_1 along the line with an upward trend to the right, and in the downward direction are respectively a_4, b_4 and c_4. Let the differences between p_7 and p_6, p_6 and p_5, etc., be denoted by m, then

$$m\,dv / dp = a_3 + 1 / 2\,(b_2) + 1 / 3\,(c_1)$$

and

$$a_4 - 1 / 2\,(b_4) + 1 / 3\,(c_4),$$

both series being continued till an order of differences is reached that vanishes. Then

$$2\,m\,dv / dp = (a_3 + a_4) - 1 / 2\,(b_4 - b_2) - 1 / 3\,(c_4 + c_1),$$

etc. The first, second and third approximations are the first quantity—the first two and the first three.

The proper second approximation is:

$$2\,m\,dv / dp = (a_3 + a_4) - 1 / 6\,(b_4 - b_2).$$

This would be exact if v were expressed by equation:

$$v = ap + bp^2 + cp^3 + dp^4.$$

The second equation above is obtained by using Taylor's formula:

$$u_x = u_0 + xa_0 + \frac{x^2}{2}\,b_0 + \frac{x\,(x^2 - 1)}{3}\,c_0 + \frac{x^2\,(x^2 - 1)}{4}\,d_0,$$

differentiating with regard to x and putting $x = o$:

$$du / dx_0 = a_0 - 1 / 6\,c_0 + 1 / 30\,e_0, \text{ etc.}$$

Since only two differences can be obtained that are sufficiently accurate for use, it is not worth while to consider

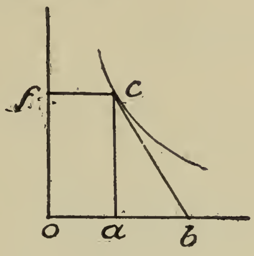

FIG. 1.

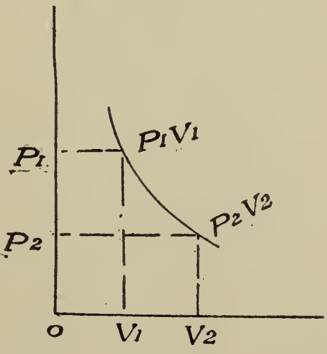

FIG. 2.

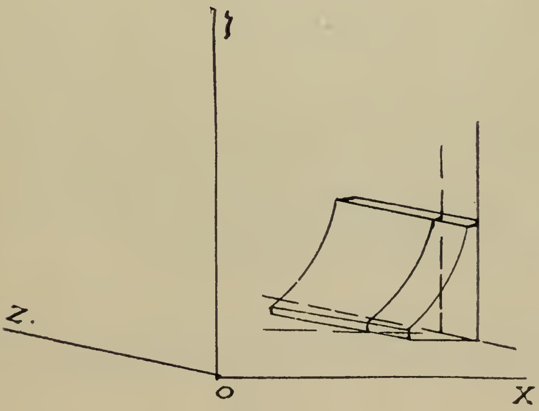

FIG. 3.

the third approximation. So, if from the adiabatics, *Figs. 4, 5, 6* and *7*, a table be constructed of pressures, volumes, first and second differences, the values of dv/dp may be obtained. Taking the reciprocal of dv/dp and substituting in the formula $dp/dv \div p/v = n$, the instantaneous values of the exponent may be found.

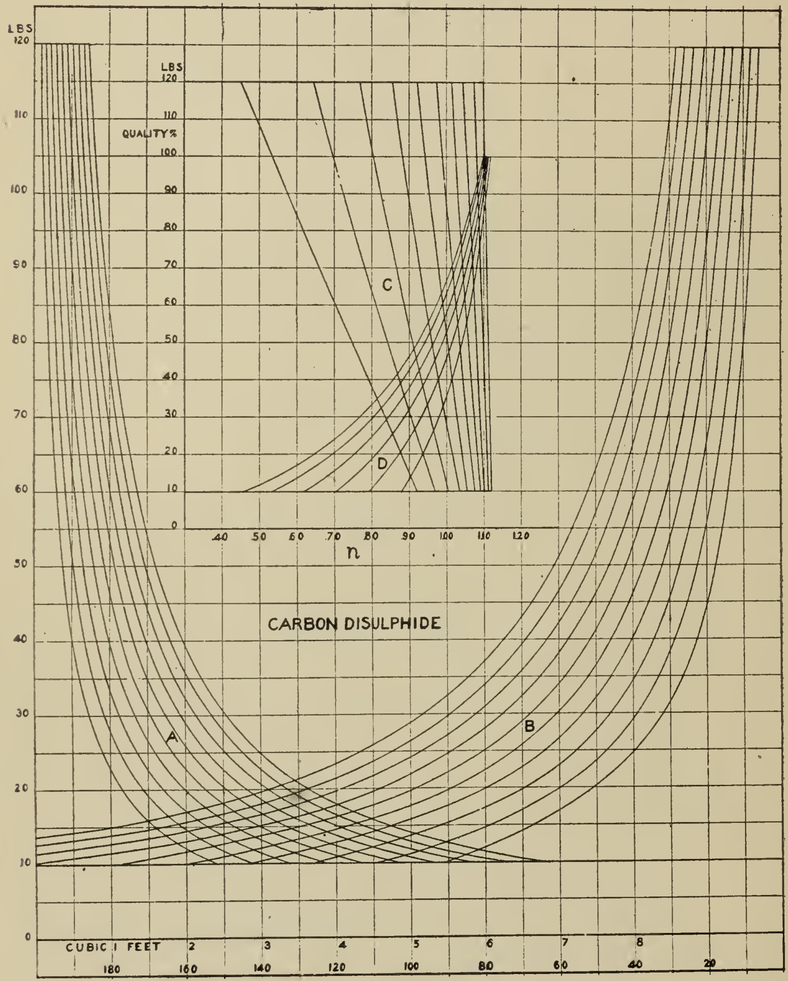

FIG. 4.

Graphical Methods.—In *Fig. 1* let *c* be the point on the adiabatic for which the exponent is desired. Take value

cf ($= oa$). Place a straight edge tangent at c, and read ob, then $oa \div (ob - ab) = n$; for since

$$n = dy / dx \div y / x = ca / ab \div ca / oa = oa / ab = n.$$

Note.—Even with extreme care in drawing tangent cb the error in the value of n may be as much as 10 per cent.

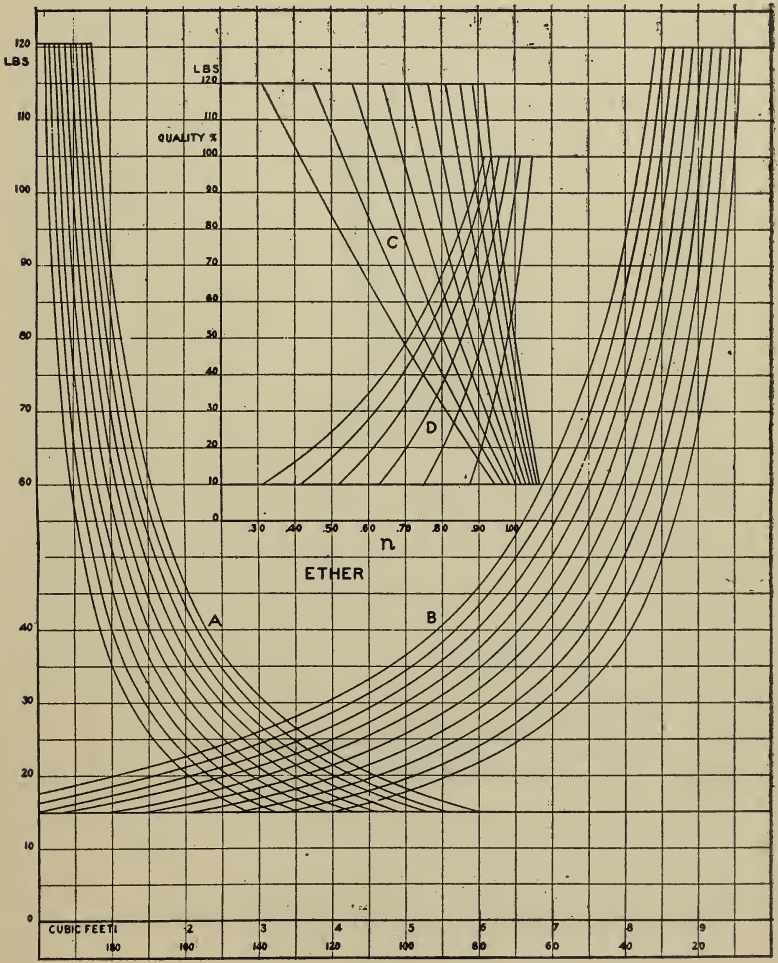

Fig. 5.

To obtain an average value of n.—Fig. 2—On the *Y* axis of logarithmic cross-section paper lay off of p_1 and p_2 from *O*. On the *X* axis do likewise for v_1 and v_2. On either axis lay off $p_1 - p_2$ and and $v_2 - v_1$ from *O*. Take difference $(p_1 - p_2) -$

$(v_2\ v_1)$ with a pair of compasses and determine the exponent by measuring from *O* and reading the number corresponding.

It has just been stated that Rankine's value of *n* for adiabatic expansion of steam was 10/9, or 1·11. This was

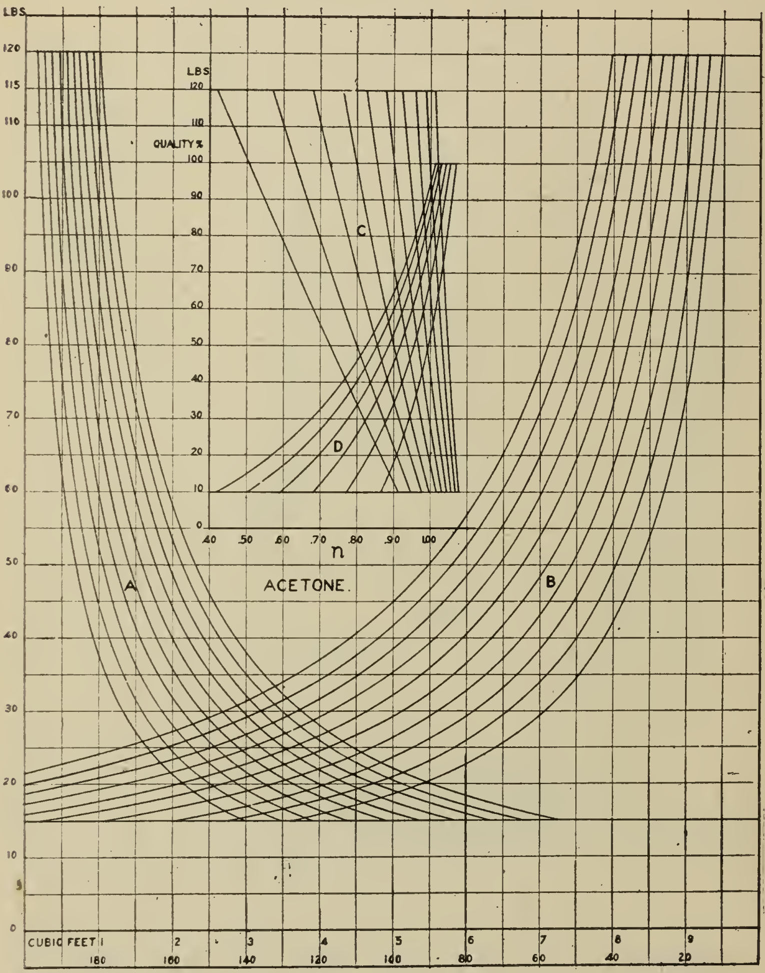

FIG. 6.

later pointed out by Grashof to be too small, but was true for initial dryness fraction of between 75 and 80 per cent. 76 per cent., that is, for a definite mixture of steam and water, and that for expansion from an initial state of perfect dryness, $n = 1.14$.

Zeuner made calculations, using initial pressures of 4, 2 and 1 atmospheres, and final pressures of 2, 1 and 5/10, 1 and 5/10, and 5/10 atmospheres, obtaining the following exponents:

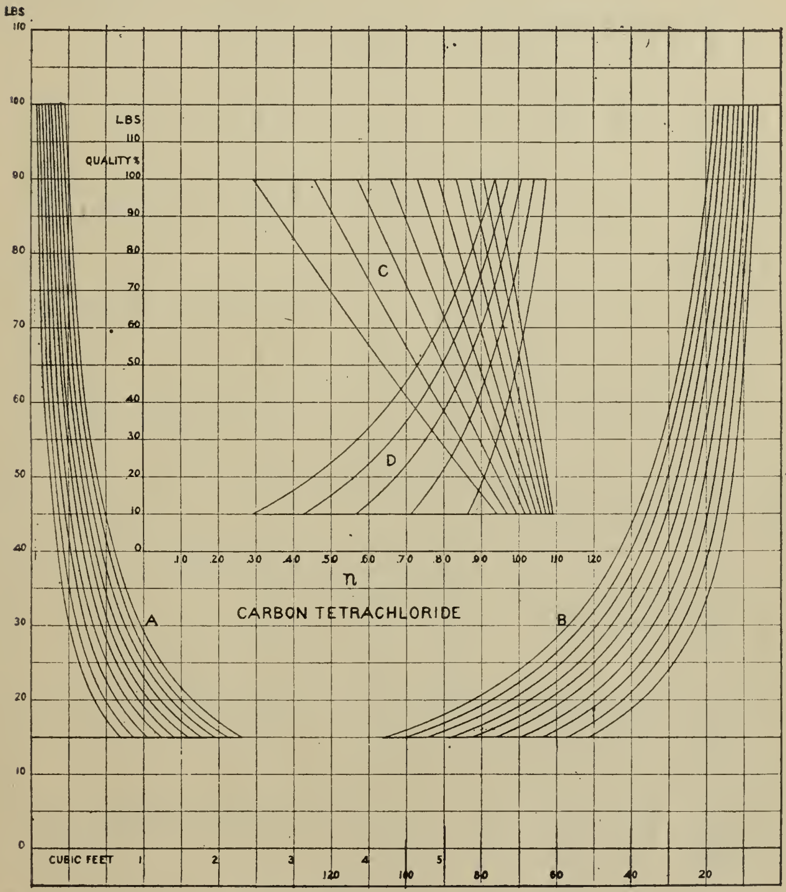

FIG. 7.

$x = 100$ per cent. $n = 1{\cdot}135$
$x = 90$ " $n = 1{\cdot}125$
$x = 80$ " $n = 1{\cdot}115$
$x = 70$ " $n = 1{\cdot}103$

and finds that for adiabatic expansion of steam whose dryness fraction is between 100 per cent. and 70 per cent., that the value of the exponent is fairly represented by

$$n = 1{\cdot}035 + x/10.$$

STEAM.

Exponents for Adiabatic Expansion. Average Values of "n" for Initial Qualities of

Temperature Range, Deg. F.	Pressure Range, Lbs. Abs.	100 Per Cent.	90 Per Cent.	80 Per Cent.	70 Per Cent.	60 Per Cent.	50 Per Cent.	40 Per Cent.	30 Per Cent.	20 Per Cent.	10 Per Cent.
341 to 228	119·9 to 20·	1·1397	1·1301	1·1185	1·1043	1·0863	1·0630	1·0311	·9854	·91315	·78296
293 " 194	60·45 " 10·2	1·1366	1·1273	1·1175	1·1052	1·0896	1·0691	1·0409	·9995	·93275	·80450
250 " 162	29·86 " 4·96	1·1309	1·1238	1·1151	1·1043	1·0905	1·0723	1·0469	1·0092	·94704	·82378
213 " 135	14·99 " 2·53	1·1291	1·1229	1·1152	1·1057	1·0934	1·0771	1·0542	1·0196	·96170	·84282
Average . .		1·1341	1·126	1·1166	1·1049	1·09	1·0704	1·0433	1·0034	·93866	·81352

Instantaneous Values of "n" for Initial Qualities of

Pressures, Lbs. Abs.	100 Per Cent.	90 Per Cent.	80 Per Cent.	70 Per Cent.	60 Per Cent.	50 Per Cent.	40 Per Cent.	30 Per Cent.	20 Per Cent.	10 Per Cent.
110	1·164	1·151	1·1135	1·117	1·096	1·065	1·019	·958	·854	·657
100	1·159	1·147	1·132	1·113	1·092	1·062	1·019	·960	·864	·680
90	1·153	1·142	1·128	1·110	1·090	1·060	1·019	·965	·874	·703
80	1·149	1·138	1·125	1·107	1·088	1·059	1·021	·970	·884	·728
70	1·145	1·135	1·123	1·105	1·088	1·060	1·025	·976	·897	·754
60	1·141	1·132	1·120	1·104	1·086	1·061	1·030	·983	·908	·783
50	1·138	1·130	1·118	1·103	1·086	1·063	1·035	·990	·921	·807
40	1·134	1·126	1·114	1·102	1·086	1·064	1·039	·998	·936	·836
30	1·130	1·122	1·111	1·100	1·085	1·065	1·042	1·009	·952	·867
20	1·125	1·116	1·107	1·097	1·084	1·067	1·045	1·020	·970	·879

The exponent n is practically constant in the 60 per cent. column.

CARBON DISULPHIDE.

Exponents for Adiabatic Expansion. Average Values of "n" for Initial Qualities of

Temperature Range, Deg. F.	Pressure Range, Lbs. Abs.	100 Per Cent.	90 Per Cent.	80 Per Cent.	70 Per Cent.	60 Per Cent.	50 Per Cent.	40 Per Cent.	30 Per Cent.	20 Per Cent.	10 Per Cent.
265 to 135	120·3 to 20·7	1·1127	1·0963	1·0770	1·0542	1·0262	·9922	·9488	·8913	·7338	·6889
205 " 95	58·55 " 10·4	1·1268	1.1131	1·0969	1·0778	1·0554	1·0217	·9916	·9273	·8278	·7166
160 " 60	30·96 " 4·82	1·1294	1·1179	1·1042	1·0912	1·0600	1·0399	1·0042	·9542	·8482	·7476
115 " 32	14·57 " 2·47	1·1344	1·1246	1·1129	1·0984	1·0802	1·0564	1·0242	·9779	·9052	·7727

Instantaneous Values of "n" for Initial Qualities of

Pressures, Lbs. Abs.	100 Per Cent.	90 Per Cent.	80 Per Cent.	70 Per Cent.	60 Per Cent.	50 Per Cent.	40 Per Cent.	30 Per Cent.	20 Per Cent.	10 Per Cent.
120	1·100	1·075	1·044	1·014	·9750	·922	·857	·769	·646	·453
110	1·101	1·078	1·049	1·020	·9815	·933	·870	·789	·670	·494
100	1·102	1·082	1·053	1·024	·9890	·943	·883	·804	·697	·536
90	1·105	1·085	1·060	1·030	·9970	·954	·893	·825	·725	·577
80	1·107	1·089	1·066	1·037	1·007	·966	·913	·845	·753	·620
70	1·109	1·093	1·070	1·043	1·017	·978	·929	·866	·782	·660
60	1·110	1·096	1·077	1·050	1·026	·990	·945	·888	·813	·705
50	1·112	1·098	1·080	1·058	1·034	1·003	·961	·910	·843	·750
45	1·113	1·100	1·082	1·062	1·039	1·009	·970	·921	·859	·770
40	1·114	1·101	1·084	1·065	1·043	1·015	·978	·933	·874	794
35	1·115	1·103	1·087	1·070	1·048	1·021	·988	·945	·890	·815
30	1·116	1·106	1·090	1·073	1·053	1·028	·997	·956	·905	·836
25	1·117	1·107	1·092	1·078	1·059	1·034	1·006	·968	·922	·858
20	1·118	1·110	1·095	1·081	1·063	1·041	1·015	·980	·937	·880

ETHER.

Exponents for Adiabatic Expansion. Average Value for "n" for Initial Quatities of

Temperature Range, Deg. F.	Pressure Range, Lbs. Abs.	100 Per Cent.	90 Per Cent.	80 Per Cent.	70 Per Cent.	60 Per Cent.	50 Per Cent.	40 Per Cent.	30 Per Cent.	20 Per Cent.	10 Per Cent.
230 to 110	120·31 to 19·6	·9911	·9715	·9491	·9236	·8938	·8536	·8163	·7640	·6971	·6048
180 " 80	61·9 " 10·8	1·0170	·9987	·9775	·9528	·9236	·8884	·8448	·7892	·7150	·6069
135 " 55	30·62 " 6·23	1·0359	1·0193	·9997	·9764	·9483	·9137	·8699	·8125	·7329	·6113
95 " 32	14·72 " 3·54	1·0504	1·0352	1·0172	·9957	·9693	·9362	·8940	·8356	·7527	·6202

Instantaneous Values of "n" for Initial Qualities of

Pressures, Lbs. Abs.	100 Per Cent.	90 Per Cent.	80 Per Cent.	70 Per Cent.	60 Per Cent.	50 Per Cent.	40 Per Cent.	30 Per Cent.	20 Per Cent.	10 Per Cent.
120	·917	·885	·852	·812	·764	·710	·640	·557	·450	·312
110	·925	·897	·866	·828	·781	·730	·665	·558	·490	·362
100	·934	·910	·880	·848	·800	·752	·693	·620	·530	·414
90	·945	·923	·896	·862	·820	·775	·721	·653	·592	·465
80	·956	·937	·912	·881	·842	·800	·750	·689	·614	·520
70	·970	·952	·928	·900	·864	·826	·780	·725	·658	·575
60	·984	·968	·946	·920	·887	·853	·813	·763	·704	·631
50	·998	·984	·964	·941	·913	·882	·847	·803	·751	·690
40	1·014	1·000	·984	·964	·939	·920	·882	·845	·801	·750
30	1·031	1·018	1·003	·987	·966	·959	·920	·888	·853	·810
20	1·047	1·037	1·024	1·012	·995	·980	·959	·933	·907	·875
10	1·065	1·047	1·037	1·024	1·010		·980	·957	·935	·941

ACETONE.

Exponents for Adiabatic Expansion. Average Values of "n" for Initial Qualities of

Temperature Range, Deg. F.	Pressure Range, Lbs. Abs.	100 Per Cent.	90 Per Cent.	80 Per Cent.	70 Per Cent.	60 Per Cent.	50 Per Cent	40 Per Cent.	30 Per Cent.	20 Per Cent.	10 Per Cent.
284 to 158	135·5 to 23·04	1·0413	1·0229	1·0000	·9767	·9473	·9118	·8679	·8120	·7376	·6309
230 " 122	69·5 " 11·77	1·0640	1·0479	1·0292	1·0068	·9797	·9463	·9040	·8482	·7705	·6500
176 " 86	31·2 " 5·42	—	—	—	—	—	—	—	—	—	—
140 " 50	16·6 " 2·69	1·1304	1·1182	1·1036	1·0859	1·0640	1·0640	·9993	·8483	·8723	·7431

Instantaneous Values of "n" for Initial Qualities of

Pressures, Lbs. Abs	100 Per Cent.	90 Per Cent.	80 Per Cent.	70 Per Cent.	60 Per Cent.	50 Per Cent.	40 Per Cent.	30 Per Cent.	20 Per Cent.	10 Per Cent.
120	1·010	·982	·953	·912	·869	·813	·744	·657	·540	·378
110	1·016	·991	·963	·931	·889	·840	·780	·702	·600	·440
100	1·021	·997	·970	·940	·900	·854	·798	·725	·631	·504
90	1·026	1·002	·978	·950	·912	·870	·817	·750	·663	·547
80	1·030	1·010	·985	·960	·925	·885	·838	·777	·696	·597
70	1·037	1·016	·993	·970	·638	·902	·857	·801	·730	·637
60	1·042	1·023	1·003	·979	·950	·919	·877	·827	·763	·682
50	1·047	1·031	1·012	·990	·964	·937	·900	·856	·799	·727
40	1·053	1·039	1·022	1·002	·980	·954	·921	·884	·834	·772
35	1·058	1·043	1·028	1·009	·687	·963	·933	·900	·853	·796
30	1·061	1·048	1·033	1·016	·996	·973	·945	·915	·870	·818
25	1·064	1·052	1·039	1·022	1·004	·983	·958	·930	·890	·842
20	1·068	1·057	1·044	1·030	1·012	·993	·970	·945	·907	·865
15	1·083	1·062	1·050	1·036	1·020	1·003	·982	·960	·927	·888
10	1·087	1·067	1·056	1·043	1·030	1·013	·996	·975	·945	·912

CARBON TETRACHLORIDE.

Exponents for Adiabatic Expansion. Average Values of "n" for Initial Qualities of

Temperature Range, Deg. F.	Pressure Range, Lbs. Abs.	100 Per Cent.	90 Per Cent.	80 Per Cent.	70 Per Cent.	60 Per Cent.	50 Per Cent.	40 Per Cent.	30 Per Cent.	20 Per Cent.	10 Per Cent.
320 to 176	106·6 to 16·31	·9860	·9664	·9441	·9167	·8947	·8496	·8060	·7524	·6835	·5890
266 " 158	57·95 " 12·01	1 0498	1·0315	1·0100	·9740	·9550	·9183	·8724	·8129	·7320	·6114
176 " 86	16·31 " 2·751	1·0962	1 0822	1·0656	1·0455	1·021	·9898	·9494	·8944	·7977	·6839

Instantaneous Values of "n" for Initial Qualities of

Pressures, Lbs. Abs.	100 Per Cent.	90 Per Cent.	80 Per Cent.	70 Per Cent.	60 Per Cent.	50 Per Cent.	40 Per Cent.	30 Per Cent.	20 Per Cent.	10 Per Cent.
100	·9375	·906	·872	·834	·786	·730	·659	·571	·455	·293
90	·964	·930	·894	·859	·815	·763	·697	·616	·509	·360
80	·973	·946	·916	·883	·843	795	·735	·660	·563	·428
70	·990	·966	·938	·907	·870	·827	·773	·707	·617	·496
60	1·007	·985	·960	·931	·897	·860	·813	·753	·673	·567
50	1·024	1·006	·981	·957	·926	·893	·852	·800	·729	·639
45	1·033	1 016	·993	·969	·940	·910	·873	·823	·758	·675
40	1·042	1·023	1·004	·982	·954	·923	·888	·846	·787	·711
35	1·050	1·035	1·015	·994	·969	·943	·913	·870	·816	·749
30	1·058	1·014	1·027	1·007	·984	·960	·933	·894	·845	·786
25	1 066	1·054	1·038	1·020	1·000	·979	·953	·920	·876	·825
20	1·074	1·063	1·049	1 033	1·015	·996	·973	·943	·907	·863
15	1·082	1·072	1·060	1·046	1·030	1·013	·995	·970	·938	·901
10	1·090	1·082	1·072	1·060	1·047	1·031	1·016	·994	·969	·940

The tabulated values of n, in the preceding, were computed from a well-known steam table with Vega's logarithms and are essentially the averages from 100 to 70 per cent., inclusive, as found by Zeuner. It is seen that as x decreases, n decreases. For 100, 90, and 80 per cent. qualities at the several pressures n decreases; for 60 per cent. and below it increases. At 70 per cent., n is practically constant. When the curves are plotted, as in *Fig. 9*, a crossing of each other is noticed. This cannot be said of carbon di-sulphide, acetone, ether, or carbon tetrachlorides. Ammonia and some of the other vapors have not been considered on account of lack of experimental data for specific heats of the liquids. Alcohol was omitted because of the fact that dissociation takes place during evaporation.

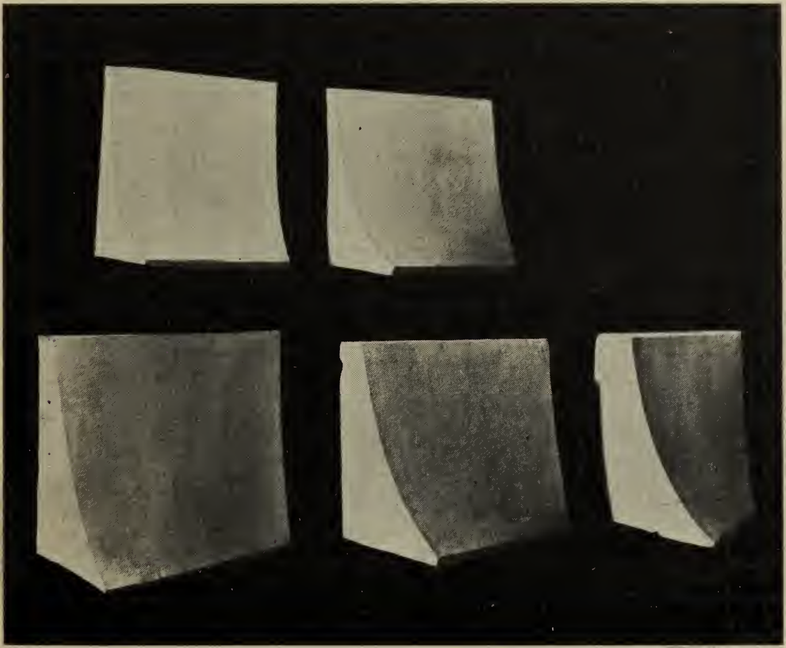

FIG. 8.

The hyperbola, which is used quite largely as a curve for comparison with the expansion curve on an indicator diagram, would not be traced by the indicator pencil, true

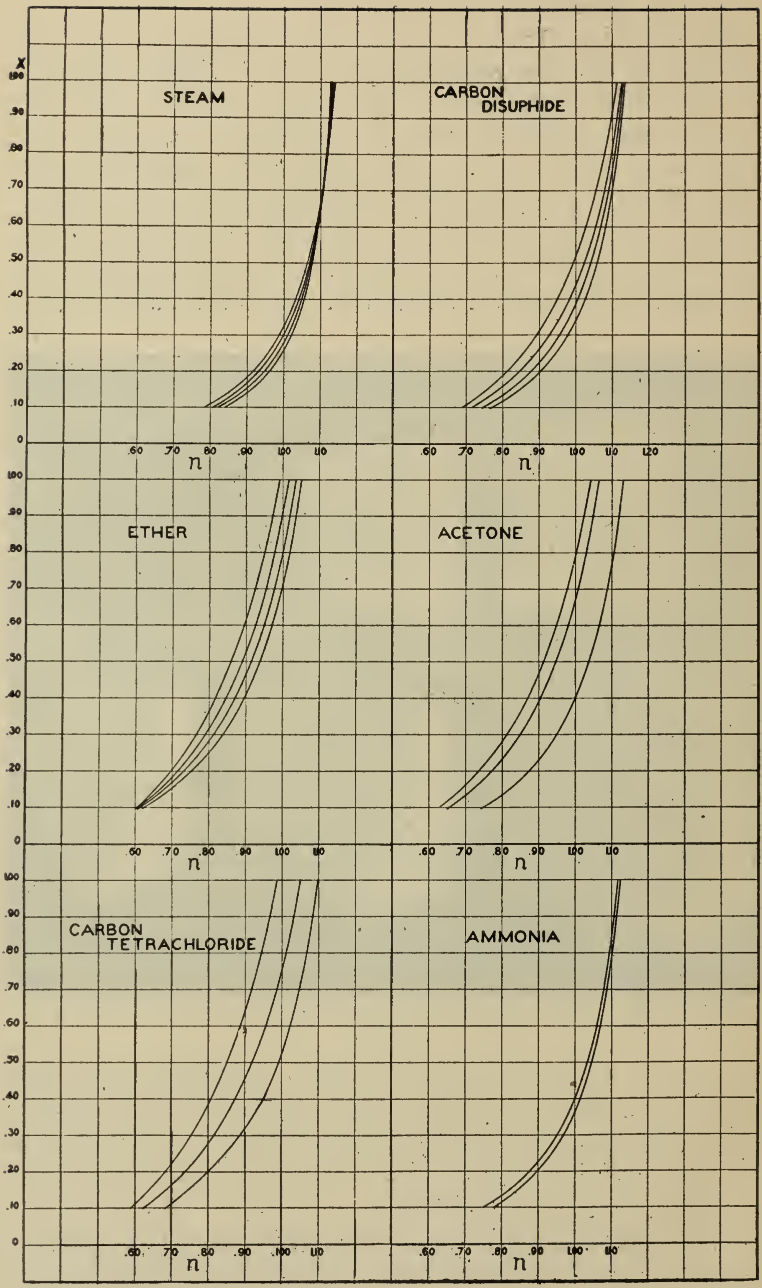
X
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
0
STEAM
CARBON DISUPHIDE
.60 .70 .80 .90 1.00 1.10
n
.60 .70 .80 .90 1.00 1.10 1.20
n
ETHER
ACETONE
.60 .70 .80 .90 1.00 1.10
n
.60 .70 .80 .90 1.00 1.10
n
CARBON TETRACHLORIDE
AMMONIA
.60 .70 .80 .90 1.00 1.10
n
.60 .70 .80 .90 1.00 1.10
n

FIG. 9.

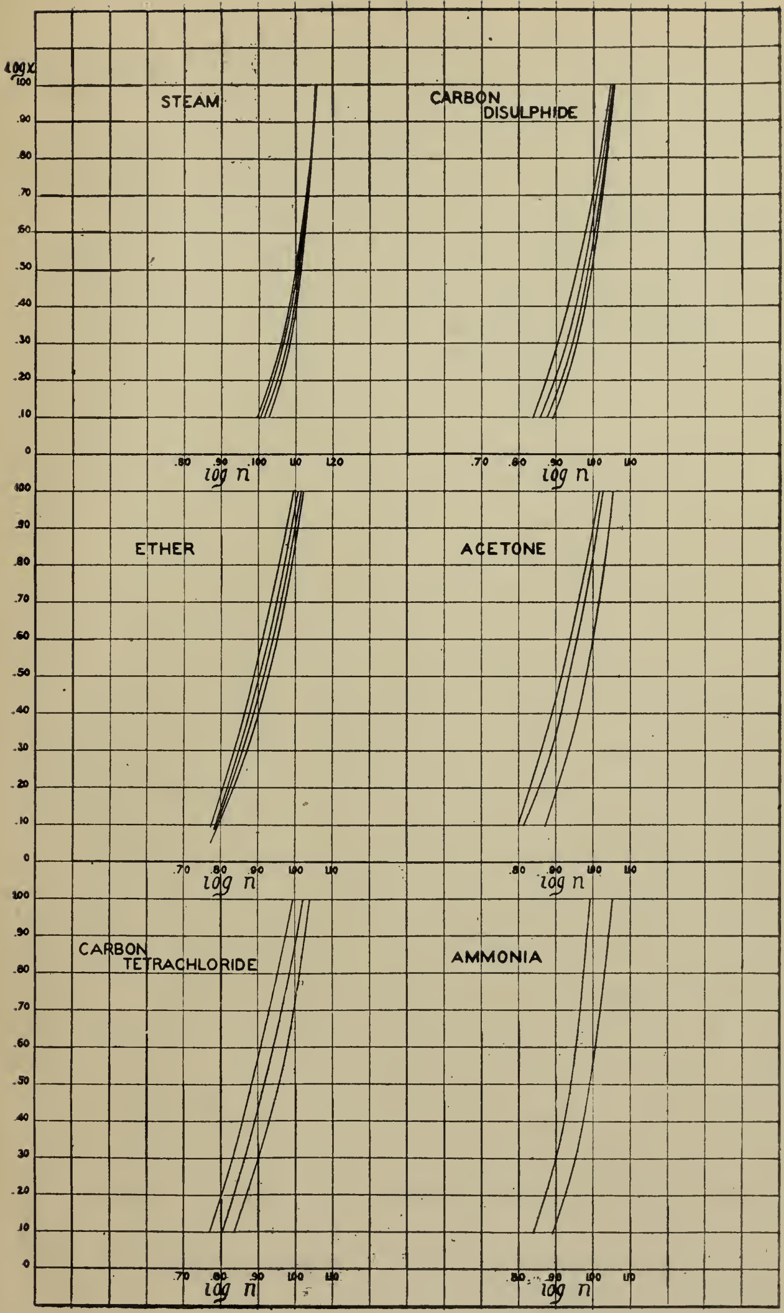

FIG. 10.—NOTE.—The characteristics of log *n* as plotted have been increased by unity.

adiabatic expansion taking place, until the quality was reduced to 31 or 32 per cent.; practically two-thirds of the weight of steam having condensed.

The lower parts of the preceding five tables give instantaneous values of n. The variation between these values and the average values above are quite noticeable.

In *Figs. 4, 5, 6* and *7*, the sets of large curves at the left are the adiabatics and those at the right, marked *B*, are their evolutes. Referring to the central diagrams, those curves which diverge upwards, *C*, give instantaneous exponent values for the different pressures along the several adiabatics, so marked from initial qualities. The other sets of curves, *D*, to the same axes, show the instantaneous values of n for any given pressure, across the several adiabatics. Since these diagrams have three sets of variables, solids may be formed which will give the instantaneous value of n for any given pressure and dryness fraction. The photograph, *Fig. 8*, is that of the five solids so obtained. Beginning at the top and reading to the right are those of steam, carbon disulphide, ether, acetone and carbon tetrachloride. Their similarity of figure is quite noticeable. If horizontal planes be passed through the surfaces, the lines of intersection are the curves *D* and *C* in the plates just described. Planes giving curves marked *D*, are those made by the *XY* planes passed through the surface, as shown in *Fig. 3*.

It was hoped that some general expression might be obtained which would give the exponents for adiabatic expansion for any vapor at any pressure with any given quality modification. It is seen that the curves *D*, *Figs. 4, 5, 6* and *7*, are not coincident, hence any expression that might be obtained would be too cumbersome to be of any practical use. *Fig. 9* shows that the average values of n do not form coincident curves. In the following *Fig. 10*, where their logarithmic values are plotted, it is seen that while the curves are not coincident they form almost straight lines to 60 per cent. quality; that is, any given curve representing the varying exponential values at a given pressure, is almost logarithmic. That being the case,

$$n = (x / c)^{a}$$

If now an expression for the 100 per cent. initial quality line of *C*, of the preceding *Figs. 4*, *5*, *6* and *7*, be obtained, which would give the value a at any given pressure, then n for a limited range becomes known.

It may be further said, upon comparing exponents for any given temperature, or any given pressure for the several vapors, or for any given range of pressure of the several vapors, it was found that there was no direct relationship between those values of n and latent, total, or specific heats.

3 0112 061413099